

LMR Geometric Identity Atlas

Length–Mass Reduction Theory

Notation Convention

Tier-1 structural quantities are written in codex grammar, using primes only where required by Paper I. When ° grammar is used, extra primes are not added unnecessarily.

Tier-3 lm-reduced SI comparison quantities are written using the explicit subscript notation:

$${}_{\text{SI}}M', \quad {}_{\text{SI}}\lambda', \quad {}_{\text{SI}}\alpha', \quad {}_{\text{SI}}E', \quad {}_{\text{SI}}\sqrt{G'}$$

If a quantity is numerically identical in SI and lm-reduced SI, no extra subscript distinction is required by codex rule.

Input Constant

All Tier-1 identities below are derived from the single physical input

$$c = 299\,792\,458 \text{ m/s}$$

together with geometric closure constants.

Perspective Conversion Box

$$\sqrt{G'} = \frac{1}{3c^3}$$

$$\ell'_G = \sqrt{\sqrt{G'}}$$

$$\sqrt{3} \ell'_G = \frac{1}{(\sqrt{c})^3}$$

$$M' = \frac{1}{\lambda}$$

$$t = \frac{\lambda}{c}$$

$$f = \frac{c}{\lambda}$$

These identities convert between the λ perspective and the M' perspective and generate the cadence relations of the Tier-1 atlas.

Cadence Ladder

$$\lambda \sim (\sqrt{c})^{-n}$$

$$t = \frac{\lambda}{c} \sim (\sqrt{c})^{-(n+2)}$$

$$f = \frac{c}{\lambda} \sim (\sqrt{c})^{n+2}$$

$$M' = \frac{1}{\lambda} \sim (\sqrt{c})^n$$

$$E' = M'c^2 \sim (\sqrt{c})^{n+4}$$

Thus each configuration occupies a fixed rung on the \sqrt{c} ladder.

1. Foundational Corridor Identities

Gravity Corridor

$$\sqrt{G'} = \frac{1}{3c^3}$$

$$\ell'_G = \sqrt{\sqrt{G'}}$$

$$\sqrt{3} \ell'_G = \frac{1}{(\sqrt{c})^3}$$

$$R_{\text{abs}} = \sqrt{G'} M' = \frac{1}{3\lambda c^3}$$

Foundational Numerical Values

$$\sqrt{c} \approx 1.7314516 \times 10^4$$

$$\sqrt{G'} \approx 1.2363665 \times 10^{-26}$$

$$\ell'_G \approx 1.1119202 \times 10^{-13}$$

$$\sqrt{3} \ell'_G \approx 1.9251086 \times 10^{-13}$$

2. Electron Configuration

Electron Configuration

λ Perspective

$$\lambda_e^\circ = \frac{4\pi}{(\sqrt{c})^3}$$

$$t_e^\circ = \frac{4\pi}{(\sqrt{c})^5}$$

$$f_e^\circ = \frac{(\sqrt{c})^5}{4\pi}$$

$$M_e^\circ = \frac{(\sqrt{c})^3}{4\pi}$$

$$E_e^\circ = M_e^\circ c^2 = \frac{(\sqrt{c})^7}{4\pi}$$

M' Perspective

$$\lambda_e^\circ = 4\pi\sqrt{3}\ell'_G$$

$$t_e^\circ = \frac{4\pi\sqrt{3}\ell'_G}{c}$$

$$f_e^\circ = \frac{c}{4\pi\sqrt{3}\ell'_G}$$

$$M_e^\circ = \frac{1}{4\pi\sqrt{3}\ell'_G}$$

$$E_e^\circ = \frac{c^2}{4\pi\sqrt{3}\ell'_G}$$

Electron Numerical Values

$$\lambda_e^\circ \approx 2.4209109 \times 10^{-12}$$

$$t_e^\circ \approx 8.0756110 \times 10^{-21}$$

$$f_e^\circ \approx 1.2382966 \times 10^{20}$$

$$M_e^\circ \approx 4.1305278 \times 10^{11}$$

$$E_e^\circ \approx 3.7114895 \times 10^{28}$$

Electron Comparison to SI

Identity	Tier-1 Value	SI / lm-Reduced SI	Percent Error
λ_e°	$2.4209109 \times 10^{-12}$	$2.4263102 \times 10^{-12}$	-0.223%
M_e°	4.1305278×10^{11}	SI M'_e	same family
f_e°	1.2382966×10^{20}	SI f'_e	same family
t_e°	$8.0756110 \times 10^{-21}$	SI t'_e	same family
E_e°	3.7114895×10^{28}	SI E'_e	same family

Hierarchy

Tier-1 establishes the electron as the minimal open half-fold certificate derived from lattice cadence. SI captures the electron through empirical measurement chains such as Penning-trap frequency, electron anomaly, and spectroscopic inference. All electron-family percent errors are expected to track together if the same realization separation is active across the configuration.

3. Proton Configuration

Proton Configuration

λ Perspective

Pre-bookkeeping:

$$\lambda_p^\circ = \frac{4\pi}{(\sqrt{c})^4}$$

$$t_p^\circ = \frac{4\pi}{(\sqrt{c})^6}$$

$$f_p^\circ = \frac{(\sqrt{c})^6}{4\pi}$$

$$M_p^\circ = \frac{(\sqrt{c})^4}{4\pi}$$

Post-bookkeeping:

$$^\circ\lambda_p = \frac{4\pi \cdot 3\pi}{(\sqrt{c})^4}$$

$$^\circ t_p = \frac{4\pi \cdot 3\pi}{(\sqrt{c})^6}$$

$$^\circ f_p = \frac{(\sqrt{c})^6}{4\pi \cdot 3\pi}$$

$$^\circ M_p = \frac{(\sqrt{c})^4}{4\pi \cdot 3\pi}$$

M' Perspective

Pre-bookkeeping:

$$\lambda_p^\circ = \frac{4\pi\sqrt{3}\ell'_G}{\sqrt{c}}$$

$$t_p^\circ = \frac{4\pi\sqrt{3}\ell'_G}{c\sqrt{c}}$$

$$f_p^\circ = \frac{c\sqrt{c}}{4\pi\sqrt{3}\ell'_G}$$

$$M_p^\circ = \frac{\sqrt{c}}{4\pi\sqrt{3}\ell'_G}$$

Post-bookkeeping:

$$^\circ\lambda_p = \frac{4\pi \cdot 3\pi \cdot \sqrt{3}\ell'_G}{\sqrt{c}}$$

$$^\circ t_p = \frac{4\pi \cdot 3\pi \cdot \sqrt{3}\ell'_G}{c\sqrt{c}}$$

$$^\circ f_p = \frac{c\sqrt{c}}{4\pi \cdot 3\pi \cdot \sqrt{3}\ell'_G}$$

$$^\circ M_p = \frac{\sqrt{c}}{4\pi \cdot 3\pi \cdot \sqrt{3}\ell'_G}$$

Proton Numerical Values

$$\lambda_p^\circ \approx 1.3981973 \times 10^{-16}$$

$$^\circ\lambda_p \approx 1.3172910 \times 10^{-15}$$

$$M_p^\circ \approx 7.1512802 \times 10^{15}$$

$$^\circ M_p \approx 7.5916444 \times 10^{14}$$

Proton Comparison to SI

Identity	Tier-1 Value	SI / lm-Reduced SI	Percent Error
$\lambda_e^\circ / ^\circ\lambda_p$	$\sqrt{c}/(3\pi) = 1837.1272$	$m_p/m_e = 1836.1527$	+0.0531%
$^\circ M_p / M_e^\circ$	1837.1272	$_{\text{SI}}M'_p / _{\text{SI}}M'_e$	same family

Hierarchy

The proton is the minimal four-half-fold reflective basin. Post-bookkeeping introduces the 3π pseudo-closure, encoding triadic facing multiplicity and admissibility orientation. The electron-proton ladder is one of the principal Tier-1 versus SI anchors.

4. Hydrogen Configuration

Hydrogen Configuration

λ Perspective

$$\lambda_H^\circ = \frac{\lambda_p^\circ}{\sqrt{\pi}} = \frac{4\pi}{\sqrt{\pi}(\sqrt{c})^4}$$

$$t_H^\circ = \frac{\lambda_H^\circ}{c} = \frac{4\pi}{\sqrt{\pi}(\sqrt{c})^6}$$

$$f_H^\circ = \frac{c}{\lambda_H^\circ} = \frac{\sqrt{\pi}(\sqrt{c})^6}{4\pi}$$

$$M_H^\circ = \frac{1}{\lambda_H^\circ} = \frac{\sqrt{\pi}(\sqrt{c})^4}{4\pi}$$

M' Perspective

$$\lambda_H^\circ = \frac{4\pi\sqrt{3}\ell'_G}{\sqrt{\pi}\sqrt{c}}$$

$$t_H^\circ = \frac{4\pi\sqrt{3}\ell'_G}{c\sqrt{\pi}\sqrt{c}}$$

$$f_H^\circ = \frac{c\sqrt{\pi}\sqrt{c}}{4\pi\sqrt{3}\ell'_G}$$

$$M_H^\circ = \frac{\sqrt{\pi}\sqrt{c}}{4\pi\sqrt{3}\ell'_G}$$

Hydrogen Numerical Values

$$\lambda_H^\circ \approx 7.8876441 \times 10^{-17}$$

$$t_H^\circ \approx 2.6303596 \times 10^{-25}$$

$$f_H^\circ \approx 3.8017626 \times 10^{24}$$

$$M_H^\circ \approx 1.2678058 \times 10^{16}$$

Hydrogen Notes

Hydrogen is satiation, not pseudo-closure. Paper III defines

$$\lambda_H^\circ = \frac{\lambda_p^\circ}{\sqrt{\pi}}$$

but does **not** define a post-bookkeeping $^\circ\lambda_H$. Hydrogen is therefore treated as schedule-bearing and likely manifests as a band / family of admissible releases rather than a single locked spectral notch.

Hierarchy

Hydrogen temporarily satisfies the open half-fold until cadence exhaustion and reseating. It lies structurally between proton seating and neutron torsion compression. Its spectral realization likely introduces the distinction between geometric and observed Rydberg scales.

5. Neutron Configuration

Neutron Configuration

λ Perspective

Pre-bookkeeping:

$$\lambda_n^\circ = \frac{\lambda_p^\circ}{\pi} = \frac{4\pi}{\pi(\sqrt{c})^4}$$

Post-bookkeeping:

$$^\circ\lambda_n = \frac{4\pi \cdot 3\pi \cdot 3}{\pi(\sqrt{c})^4}$$

$$^\circ t_n = \frac{4\pi \cdot 3\pi \cdot 3}{\pi(\sqrt{c})^6}$$

$$^\circ f_n = \frac{\pi(\sqrt{c})^6}{4\pi \cdot 3\pi \cdot 3}$$

$$^\circ M_n = \frac{\pi(\sqrt{c})^4}{4\pi \cdot 3\pi \cdot 3}$$

M' Perspective

Pre-bookkeeping:

$$\lambda_n^\circ = \frac{4\pi\sqrt{3}\ell'_G}{\pi\sqrt{c}}$$

Post-bookkeeping:

$$^\circ\lambda_n = \frac{4\pi \cdot 3\pi \cdot 3 \cdot \sqrt{3}\ell'_G}{\pi\sqrt{c}}$$

$$^\circ t_n = \frac{4\pi \cdot 3\pi \cdot 3 \cdot \sqrt{3}\ell'_G}{\pi c\sqrt{c}}$$

$$^\circ f_n = \frac{\pi c\sqrt{c}}{4\pi \cdot 3\pi \cdot 3 \cdot \sqrt{3}\ell'_G}$$

$$^\circ M_n = \frac{\pi\sqrt{c}}{4\pi \cdot 3\pi \cdot 3 \cdot \sqrt{3}\ell'_G}$$

Neutron Numerical Values

$$\lambda_n^\circ \approx 4.4505994 \times 10^{-17}$$

$$^\circ\lambda_n \approx 4.1913813 \times 10^{-16}$$

$$^\circ M_n \approx 2.3850833 \times 10^{15}$$

$$T_n = \frac{\sqrt{c}}{2\pi^2} \approx 8.7698342 \times 10^2$$

Key Neutron Ratios

$$\frac{^\circ\lambda_p}{^\circ\lambda_n} = \frac{\pi}{3}$$

$$\frac{^\circ M_p}{^\circ M_n} = \frac{3}{\pi}$$

$$\frac{\lambda_e^\circ}{^\circ\lambda_n} = \frac{\sqrt{c}}{9}$$

Hierarchy

The neutron is pseudo-closure: torsion-locked hydrogen seating. The capstone token $3/\pi$ indicates terminal locking — nowhere left to go, no tighter single configuration, and no internal schedule except rarefaction after support loss. The delay time

$$T_n = \frac{\sqrt{c}}{2\pi^2}$$

is interpreted as the unsupported redistribution interval.

6. Interaction Identities

Known Interaction Identities

$$t_e^\circ = \frac{\lambda_p^\circ}{\sqrt{c}}$$

$$f_e^\circ = \frac{\sqrt{c}}{\lambda_p^\circ}$$

$$\frac{\lambda_e^\circ}{\circ\lambda_p} = \frac{\sqrt{c}}{3\pi}$$

$$\frac{\circ\lambda_p}{\circ\lambda_n} = \frac{\pi}{3}$$

$$\frac{\lambda_e^\circ}{\circ\lambda_n} = \frac{\sqrt{c}}{9}$$

$$\frac{\circ M_p}{\circ M_n} = \frac{3}{\pi}$$

$$\lambda_H^\circ = \frac{\lambda_p^\circ}{\sqrt{\pi}}$$

$$\lambda_n^\circ = \frac{\lambda_p^\circ}{\pi}$$

Interaction Numerical Values

$$\frac{\sqrt{c}}{3\pi} \approx 1837.1272$$

$$\frac{\pi}{3} \approx 1.0471976$$

$$\frac{3}{\pi} \approx 0.9549297$$

$$\frac{\sqrt{c}}{9} \approx 1923.8356$$

$$\sqrt{\pi} \approx 1.7724539$$

Hierarchy

These relations form the core Tier-1 interaction ladder linking electron, proton, hydrogen, and neutron.

- \sqrt{c} records cadence / half-fold depth mismatch.
- 3π records proton pseudo-closure with triadic facing multiplicity.
- $\pi/3$ and $3/\pi$ record neutron capstone termination.
- $\sqrt{\pi}$ records hydrogen satiation restraint.

These are structural identities, not force laws or dynamical transitions.

7. Structural Tokens and Closures

Structural Tokens

Token	Structural Meaning
4π	full closure / loop closure
π	admissibility / orientation bookkeeping
$\sqrt{\pi}$	hydrogen satiation restraint
$\sqrt{3}$	three-dimensional closure / volumetric token
ℓ'_G	gravity-proxy length (proxy only)
3	triadic interface multiplicity
$9 = 3 \times 3$	triadic ² multiplicity
3π	proton pseudo-closure
$3/\pi$	neutron capstone termination
$\pi/3$	neutron compression ratio in λ -perspective
\sqrt{c}	cadence / half-fold depth
2π	minimal return-to-same-state cadence

Token Discipline

- $4\pi, \pi, \sqrt{\pi}, \sqrt{3}, 3, \pi/3, 3/\pi$ are **tokens / closures / engagement factors**.
- They are **not** new corridors.
- ℓ'_G is a **proxy shorthand only**, never a primitive corridor.
- Primitive corridor quantities remain restricted to the locked set.

Perspective Bridge Identity

$$\sqrt{3} \ell'_G = \frac{1}{(\sqrt{c})^3}$$

This identity converts the M' -perspective expressions into the λ -perspective expressions and demonstrates the internal closure of the Tier-1 geometry.

8. Structural Timescales

Canonical and Derived Timescales

$$t_e^\circ = \frac{\lambda_e^\circ}{c} = \frac{4\pi}{(\sqrt{c})^5}$$

$$\tau_{\text{geom}} = \frac{1}{cR'_{\text{geom}}} = 8t_e^\circ$$

$$\tau_R = \frac{1}{cR_\infty}$$

$$T_n = \frac{\sqrt{c}}{2\pi^2}$$

Timescale Numerical Values

$$t_e^\circ \approx 8.0756110 \times 10^{-21}$$

$$\tau_{\text{geom}} = 8t_e^\circ \approx 6.4604888 \times 10^{-20}$$

$$\tau_R = \frac{1}{cR_\infty} \approx 3.0373154 \times 10^{-16}$$

$$T_n \approx 8.7698342 \times 10^2$$

Interpretation

- t_e° : canonical electron cadence unit.
- τ_{geom} : minimal geometric hydrogen schedule scaffold.
- τ_R : realized hydrogen spectral cadence scale.
- T_n : neutron redistribution delay / unsupported torsion-release interval.

These are distinct structural clocks and should not be conflated.

9. Geometric and Spectral Rydberg

Rydberg Separation

Tier-3 lm-reduction gives

$$R_\infty = \frac{1}{2} \alpha^2 M_e^\circ$$

Define the geometric Rydberg scale:

$$R'_{\text{geom}} = \frac{M_e^\circ}{8}$$

Define the realization constant:

$$\Xi_R = \frac{R'_{\text{geom}}}{R_\infty}$$

Then the spectral coupling is

$$\alpha'_{\text{spec}} = \frac{1}{2\sqrt{\Xi_R}}.$$

Rydberg Numerical Values

$$R'_{\text{geom}} = \frac{M_e^\circ}{8} \approx 5.1631598 \times 10^{10}$$

$$R_\infty \approx 1.0973732 \times 10^7$$

$$\Xi_R \approx 4705.1868$$

$$\alpha'_{\text{spec}} \approx 0.00728923$$

Hierarchy

LMR separates:

- a **bare geometric Rydberg** R'_{geom} ,
- an **observed spectral Rydberg** R_{∞} ,
- and a **realization constant** Ξ_R linking them.

This separation is one of the clearest current clues that Tier-1 and Tier-3 are parallel but not identical.

10. Promising but Not Locked

Promising Candidates

$$q' = \frac{1}{9c^2}$$

$$X' = \frac{q'}{f_e^{\circ}} = \frac{4\pi}{9c^{9/2}}$$

$$M_e^{\circ} X' = \sqrt{\varepsilon'_0}$$

$$M_e^{\circ} X' c = q'$$

$$X' E_e^{\circ} = \frac{1}{9c}$$

$$K_0 = \frac{8\sqrt{c}}{3\pi^2}$$

$$\alpha'_{\text{closure}} = \frac{1}{2}$$

$$\alpha'_{\text{spec}} = \frac{1}{2\sqrt{\Xi_R}}$$

Promising Numerical Values

$$q' \approx 1.2368180 \times 10^{-18}$$

$$X' \approx 9.9884706 \times 10^{-39}$$

$$K_0 \approx 4678.2060$$

$$\frac{\Xi_R}{K_0} \approx 1.0057673$$

Status Note

These relations are structurally promising and internally coherent, but they are **not yet locked** into the codex.

In particular:

- q' and X' are to be tested as candidate Tier-3 / EM-layer outputs,
- K_0 is currently the leading structural candidate for the Rydberg realization constant,
- α'_{closure} and α'_{spec} are to be kept distinct unless a direct derivation is established.

11. Master Percent Error Ledger

Master Error Ledger

Identity	Tier-1 Value	SI Target	Percent Error
λ_e°	$2.4209109 \times 10^{-12}$	$2.4263102 \times 10^{-12}$	-0.223%
$\lambda_e^\circ / \lambda_p$	1837.1272	1836.1527	+0.0531%
λ_p / λ_n	$\pi/3 = 1.0471976$	$m_p/m_n \approx 0.998624$	+4.87%
R'_{geom}	5.1631598×10^{10}	$R_\infty = 1.0973732 \times 10^7$	large separation
K_0	4678.2060	$\Xi_R = 4705.1868$	-0.5767%
α'_{spec}	0.00728923	0.00729735	-0.1113%

Ledger Note

The master ledger is intended to diagnose whether the mismatch between Tier-1 geometry and Tier-3 realization is:

- constant,
- oscillatory,
- convergent,
- or governed by a small family of realization constants.

The current data suggest that the mismatch is not random and may cluster around a small number of structural realization factors.

12. Closing Summary

Summary

This atlas records the parallel expressions of Tier-1 geometric identities in λ -perspective and M' -perspective.

The principal locked structures now include:

- the \sqrt{c} cadence ladder,
- the gravity corridor

$$\sqrt{G'} = \frac{1}{3c^3},$$

- the proxy bridge

$$\sqrt{3} \ell'_G = \frac{1}{(\sqrt{c})^3},$$

- electron as open certificate,
- proton as 4HF reflective basin,
- hydrogen as satiation,
- neutron as pseudo-closure,
- capstone termination

$$\frac{3}{\pi},$$

- and the separation between geometric and realized spectral scales.

The remaining unresolved task is to determine the exact nature of the realization mismatch between Tier-1 and Tier-3 and whether it is governed by one or more universal realization constants.