The Shared Dimensional Backbone

A Tier 3 Demonstration Note

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Purpose: This note places the major equations of modern physics side by side after ℓ_m reduction to demonstrate that they share a common kg-free dimensional skeleton. This is a Tier 3 (dimensional correspondence) result. It does not invoke Tier 1 structural ontology or Tier 2 spectral overlays.

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1. The Bridge

A single substitution isolates the kilogram:

$$\ell_m = \frac{h}{c}, \qquad m = \ell_m M', \qquad M' = \frac{1}{\lambda}$$

The bridge quantity ℓ_m carries units [kg · m] and absorbs all kilogram dependence. After substitution, $\hbar = \ell_m c/(2\pi)$ and $mc/\hbar = 2\pi M'$. No physical content is altered. Only the dimensional packaging changes.

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2. The Reduced Equations

2.1 Quantum Mechanics: Schrödinger Equation

SI form:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi$$

ℓ_m-reduced form:

$$i\frac{c}{2\pi}\frac{\partial \psi}{\partial t} = -\frac{c^2}{8\pi^2 M'}\nabla^2\psi + V'\psi$$

where $V' = V/\ell_m$.

Surviving dimensional content: c, $M' = 1/\lambda$, and 2π (closure factor).

What was removed: $\hbar$ (absorbed into $\ell_m c/(2\pi)$), m (replaced by $\ell_m M'$).

2.2 Relativistic Quantum Mechanics: Klein–Gordon Equation

SI form:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m^2 c^2}{\hbar^2}\right)\psi = 0$$

ℓ_m-reduced form:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + 4\pi^2 M'^2\right)\psi = 0$$

Surviving dimensional content: c, M′, and $4\pi^2 = (2\pi)^2$ (closure factor squared).

What was removed: $\hbar$ and m, via $m^2c^2/\hbar^2 = 4\pi^2M'^2$.

2.3 Relativistic Quantum Field Theory: Dirac Equation

SI form:

$$(i\hbar\gamma^\mu\partial_\mu - mc)\psi = 0$$

ℓ_m-reduced form (divide through by $\hbar$ first):

$$(i\gamma^\mu\partial_\mu - 2\pi M')\psi = 0$$

Surviving dimensional content: c (implicit in $\gamma^\mu \partial_\mu$ metric), M′, and 2π.

What was removed: $\hbar$ and m, via $mc/\hbar = 2\pi M'$. The γ matrices are dimensionless and unchanged.

2.4 Relativistic Dynamics: Energy–Momentum Invariant

SI form:

$$E^2 = p^2 c^2 + m^2 c^4$$

ℓ_m-reduced form:

$$E'^2 = P'^2 c^2 + M'^2 c^4$$

with the reduced operator map $E' \to i(c/2\pi)\partial/\partial t$, $P' \to -i(c/2\pi)\nabla$.

Surviving dimensional content: c and M′.

What was removed: The kilogram, via $E = E'\ell_m$, $p = P'\ell_m$, $m = M'\ell_m$.

2.5 General Relativity: Einstein Field Equation

SI form:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

ℓ_m-reduced form:

$$G_{\mu\nu} = \frac{8\pi G'}{c^4} T'_{\mu\nu}$$

where $G' := G \ell_m$ and $T'_{\mu\nu} := T_{\mu\nu} / \ell_m$.

Surviving dimensional content: c, G′ (which carries $[m^2 s^{-1}]$ after kg removal), and $T'_{\mu\nu}$ (kg-free stress-energy).

What was removed: The kilogram from both G and $T_{\mu\nu}$, via the substitutions $G \to G'/\ell_m$ and $T_{\mu\nu} \to T'_{\mu\nu} \ell_m$. The Einstein tensor $G_{\mu\nu}$ is purely geometric and carries no kg dependence.

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3. The Side-by-Side Comparison

Equation	SI mass insertion	ℓ_m-reduced insertion	Surviving skeleton
Schrödinger	$\hbar$, m	c/(2π), M′	c, M′, 2π
Klein–Gordon	$m^2c^2/\hbar^2$	$4\pi^2 M'^2$	c, M′, 2π
Dirac	$mc/\hbar$	2πM′	c, M′, 2π
Relativistic invariant	m	M′	c, M′
Einstein field eq.	G, $T_{\mu\nu}$	G′, $T'_{\mu\nu}$	c, G′, $T'_{\mu\nu}$ (all kg-free)

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4. What Is Shared

After ℓ_m reduction, the quantum equations (Schrödinger, Klein–Gordon, Dirac) and the relativistic invariant all share a common dimensional skeleton:

c (the lattice propagation baseline) and **M′ = 1/λ** (inverse structural length).

The mass insertion $mc/\hbar$ universally reduces to $2\pi M'$ across all quantum frameworks. The relativistic invariant reduces to the same (c, M') skeleton. No kilogram survives in any of these equations.

The Einstein field equation also becomes kg-free after the same bridge substitution, with G and $T_{\mu\nu}$ both absorbing their kg dependence through ℓ_m. The surviving gravitational skeleton is $(c, G', T'_{\mu\nu})$, which is dimensionally compatible with the quantum skeleton (c, M') — both live in meters and seconds only.

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5. What Is Merely Packaged

The ℓ_m reduction does not unify the equations. It does not show that gravity and quantum mechanics are the same theory. What it shows is narrower but precise:

The kilogram is a shared dimensional carrier that can be factored out of all major frameworks simultaneously through a single bridge quantity.

After factoring, the remaining dimensional structure in each framework depends only on meters and seconds. The specific combinations differ:

- Quantum mechanics uses c and M' with closure factor 2π
- Gravity uses c and G' with stress-energy $T'_{\mu\nu}$
- The relativistic invariant uses c and M' without additional structure

These are not identical reduced equations. They are equations that share a common dimensional regime — the kg-free (m, s) skeleton — but retain distinct structural content within that regime.

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6. What This Does Not Prove About Quantum Gravity

This note does not claim:

- That quantum mechanics and general relativity are unified
- That the quantum gravity problem is solved
- That the kg-free skeleton constitutes a theory of quantum gravity
- That ℓ_m reduction produces new physical predictions

This note does claim:

- That the dimensional incompatibility conventionally attributed to the "quantum gravity problem" is at least partly an artifact of the kilogram's redundant presence in both frameworks
- That removing the kg through ℓ_m reveals a shared (m, s) backbone that was always present but obscured by dimensional packaging
- That the obstruction to combining QM and GR may not be structural but representational — the two frameworks may resist combination not because they describe incompatible physics, but because they carry the same dimensional carrier (kg) in incompatible packaging

The strongest honest statement is:

Under ℓ_m reduction, the dimensional obstruction between quantum mechanics and general relativity is removed. Whether the remaining structural differences between the reduced frameworks can also be resolved is an open question that ℓ_m reduction alone does not answer.

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7. The Reduced Operator Dictionary

For reference, the complete reduced operator map:

SI quantity	Reduced form	Surviving dimensions
m (mass)	$M' = 1/\lambda$	m^{-1}
$\hbar$	$\ell_m c/(2\pi)$	(absorbed)
$mc/\hbar$	$2\pi M'$	m^{-1}
$m^2c^2/\hbar^2$	$4\pi^2 M'^2$	m^{-2}
E (energy)	$E' = E/\ell_m$	$m\ s^{-2}$
p (momentum)	$P' = p/\ell_m$	$m^{-1}\ s^{-1}$
G	$G' = G\ell_m$	$m^2\ s^{-1}$
$T_{\mu\nu}$	$T'_{\mu\nu} = T_{\mu\nu}/\ell_m$	(kg-free)
$\hat{E}$ (operator)	$i(c/2\pi)\partial/\partial t$	—
$\hat{p}$ (operator)	$-i(c/2\pi)\nabla$	—

8. Summary

The five major equations of modern physics — Schrödinger, Klein–Gordon, Dirac, the relativistic invariant, and the Einstein field equation — all admit ℓ_m reduction to a common kg-free dimensional regime. The quantum equations share the specific insertion $2\pi M' = mc/\hbar$, while the gravitational equation shares the regime through $G' = G\ell_m$. The kilogram is the shared carrier that, once removed, reveals a common (c, λ) backbone underlying all five frameworks.

This result is dimensional, not dynamical. It removes a representational obstruction without providing a unified theory. Whether the structural differences that remain after dimensional unification can themselves be resolved is the open frontier.

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Relationship to the LMR arc: This note uses only Tier 3 (dimensional correspondence) methods. It does not depend on Papers 0–V or on any Tier 1/Tier 2 structural claims. Its results are verifiable by anyone with access to the standard SI equations and the substitution $\ell_m = h/c$.