

Tier 1 Mapping (QM GR)

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1 Internal Notes from 4-11-26

Mapping of Tier 3 Mass Representation into LMR Structure

The Tier 3 analysis establishes that both quantum and gravitational frameworks express mass through $M' = 1/\lambda$. Within the LMR structural grammar, λ corresponds to a closed routing length constructed from half-fold (HF) units.

Thus:

$$M' = \frac{1}{\lambda} \iff \textit{inverseHF} - \textit{closurelength}$$

No new primitives are introduced.

Quantum Mass Term as Internal Closure

The Klein-Gordon mass term

$$4\pi^2 M'^2$$

is quadratic in M' and independent of spatial distribution.

Within LMR structural language, this corresponds to a configuration-level closure constraint. The quadratic dependence indicates that the admissibility condition arises from pairwise or self-referential engagement within the HF inventory.

The factor $4\pi^2 = (2\pi)^2$ reflects full closure across a complete loop engagement.

Thus, the quantum mass term corresponds to:

$$\textit{internalHFclosureunderfull} - \textit{loopengagement}$$

This is independent of embedding and depends only on the configuration itself.

Gravitational Mass Term as Distributed Engagement

The reduced gravitational source term for dust,

$$\frac{M'}{V},$$

is linear in M' and explicitly dependent on spatial volume.

Within LMR structural language, this corresponds to the distribution of HF closure across an embedding region.

The quantity M'/V represents inverse-length per volume, indicating how HF closure is distributed over available embedding space.

Thus, the gravitational mass term corresponds to:

$$\textit{distributedHFengagementacrossembeddingvolume}$$

This reflects how configurations source curvature through their spatial distribution rather than internal closure alone.

Dual Representation of HF Structure

Both quantum and gravitational mass representations arise from the same underlying quantity $M' = 1/\lambda$, corresponding to inverse HF-closure length.

The distinction between the two is not in the primitive, but in its mode of realization:

- Quantum framework:

$$M'^2 \longrightarrow \textit{internalclosureofHFconfigurations}$$

- Gravitational framework:

$$\frac{M'}{V} \longrightarrow \textit{distributionofHFconfigurationsoverembedding}$$

Thus, both frameworks may be interpreted as different structural projections of the same HF inventory:

$$\textit{configurationclosure} \leftrightarrow \textit{embeddingdistribution}$$

No dynamical interpretation is implied.

Interpretive Boundary

This mapping is interpretive and belongs to LMR Tier 1.

It does not modify, derive, or extend the Tier 3 dimensional analysis. No claim is made that quantum field theory and general relativity are structurally unified within this framework.

The mapping identifies a correspondence in how a shared dimensional primitive (M') may be expressed within LMR structural language.

Quadratic vs. Linear Appearance of M'

Within the LMR structural grammar, the primitive

$$M' = \frac{1}{\lambda}$$

records inverse closure length.

The appearance of M' depends on the admissibility regime in which the configuration is read.

A quadratic appearance,

$$M'^2,$$

corresponds to internal closure evaluated against itself. This is a self-referential admissibility condition and is therefore appropriate to local configuration-level or operator-level constraints.

A linear density appearance,

$$\frac{M'}{V},$$

corresponds to closure distributed across an embedding region. This is an embedding-indexed admissibility condition and is therefore appropriate to sourced geometric response.

Thus the distinction between the two is not primitive but structural:

$$M'^2 \leftrightarrow \textit{internalself - closure}, \quad \frac{M'}{V} \leftrightarrow \textit{distributedembeddingclosure}.$$

No dynamical identification is implied.

HF Context Hypothesis for Linear and Quadratic M'

Let

$$M' = \frac{1}{\lambda}$$

denote inverse closure length.

Within the LMR structural grammar, the appearance of M' depends on the admissibility context in which a configuration is read.

If the configuration is evaluated against its own closure requirement, the closure scale appears twice:

$$\frac{1}{\lambda} \cdot \frac{1}{\lambda} = \frac{1}{\lambda^2} = M'^2.$$

This corresponds to internal self-closure or self-paired admissibility.

If the configuration is evaluated through its presentation to an embedding region, the closure scale appears once and is distributed over the available measure:

$$\frac{1/\lambda}{V} = \frac{M'}{V}.$$

This corresponds to embedding-distributed admissibility.

Thus the quadratic/linear distinction is hypothesized to arise not from different mass primitives, but from different admissibility contexts applied to the same primitive M' .

Scalar-Field Test of the M' Regime Hypothesis

Consider a real Klein–Gordon scalar field ϕ with equation

$$\left(+ \frac{m^2 c^2}{\hbar^2} \right) \phi = 0.$$

Under ℓ_m -reduction,

$$m = \ell_m M', \quad \hbar = \frac{\ell_m c}{2\pi},$$

so the mass term becomes

$$\frac{m^2 c^2}{\hbar^2} = \frac{(\ell_m M')^2 c^2}{(\ell_m c / 2\pi)^2} = 4\pi^2 M'^2.$$

Thus the reduced Klein–Gordon equation is

$$(+4\pi^2 M'^2) \phi = 0.$$

The corresponding scalar stress-energy tensor is

$$T_{\mu\nu}^{(\phi)} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left(\frac{1}{2} \partial^\alpha \phi \partial_\alpha \phi + \frac{1}{2} \frac{m^2 c^2}{\hbar^2} \phi^2 \right),$$

which reduces to

$$T_{\mu\nu}^{(\phi)} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left(\frac{1}{2} \partial^\alpha \phi \partial_\alpha \phi + 2\pi^2 M'^2 \phi^2 \right).$$

Therefore, quadratic dependence on M' is not unique to the quantum equation of motion. It also appears on the gravitational side whenever the matter source is a field self-term.

By contrast, dust stress-energy reduces as

$$T'_{\mu\nu} = \frac{M'}{V} u_\mu u_\nu,$$

which is linear in M' and explicitly distributional.

This suggests that the appearance of M' tracks admissibility regime rather than theoretical branch:

$$M'^2 \leftrightarrow \text{self-closure/fieldself-term}, \quad \frac{M'}{V} \leftrightarrow \text{distributedoccupancy/embeddingdensity}.$$

No structural identification is asserted here; this is a dimensional and classificatory observation only.

Dirac Test of the M' Regime Hypothesis

The SI Dirac equation

$$(i\hbar\gamma^\mu\partial_\mu - mc)\psi = 0$$

reduces under ℓ_m -reduction to

$$(i\gamma^\mu\partial_\mu - 2\pi M')\psi = 0,$$

since

$$\frac{mc}{\hbar} = \frac{\ell_m M' c}{\ell_m c / (2\pi)} = 2\pi M'.$$

Thus the Dirac mass insertion is linear in M' :

$$2\pi M'.$$

This contrasts with the Klein–Gordon mass term

$$4\pi^2 M'^2 = (2\pi M')^2,$$

which is quadratic, and with the dust stress-energy term

$$\frac{M'}{V},$$

which is linear but explicitly distributional.

This suggests a three-regime classification for the appearance of M' :

$$2\pi M' \leftrightarrow \textit{singleclosureinsertion},$$

$$4\pi^2 M'^2 \leftrightarrow \textit{self-pairedclosure},$$

$$\frac{M'}{V} \leftrightarrow \textit{distributedclosure}.$$

On this reading, the exponent and placement of M' track admissibility regime rather than theoretical branch.

No structural identification is asserted here; this is a dimensional and classificatory observation only.

Dirac Insertion in Dual Structural Faces

For the electron,

$$\lambda_e^\circ = \frac{4\pi}{\sqrt{c}^3}, \quad M_e'^\circ = \frac{1}{\lambda_e^\circ} = \frac{\sqrt{c}^3}{4\pi}.$$

The reduced Dirac insertion

$$2\pi M'$$

therefore admits the electron-specific form

$$2\pi M'_e = \frac{\sqrt{c}^3}{2},$$

which is the inverse-length face of the insertion.

A distinct but related closure-dressed quantity is obtained on the λ -face:

$$2\pi\lambda_e^\circ = 2\pi \cdot \frac{4\pi}{\sqrt{c}^3} = \frac{8\pi^2}{\sqrt{c}^3}.$$

These are not identical quantities. Rather, they are dual structural faces built from the same primitive closure inventory. The former records single closure insertion on the M' -face; the latter records single closure dressing on the λ -face.

Dual-Face Form of the Dirac Insertion

For any configuration x , define

$$D_x^{(M')} := 2\pi M'_x, \quad D_x^{(\lambda)} := 2\pi\lambda_x^\circ.$$

Since $M'_x = 1/\lambda_x^\circ$, these satisfy

$$D_x^{(M')} D_x^{(\lambda)} = 4\pi^2.$$

Thus the reduced Dirac insertion admits two dual structural faces:

- on the M' -face, it records single closure insertion;
- on the λ -face, it records single closure dressing.

For the electron, proton, hydrogen, and neutron, the same invariant product $4\pi^2$ is preserved, while engagement factors such as $\sqrt{\pi}$ and π redistribute between the two faces.

This suggests that the Dirac insertion $2\pi M'$ is not merely a reduced mass term, but a dual-face closure object within the LMR grammar.

No dynamical identification is asserted.

Reciprocal Migration of Engagement Factors

Let a configuration x be related to a base closure scale by an admissibility engagement factor κ_x :

$$\lambda_x^\circ = \frac{\lambda_{\text{base}}^\circ}{\kappa_x}.$$

Then, since

$$M'_x = \frac{1}{\lambda_x^\circ},$$

it follows that

$$M'_x{}^\circ = \kappa_x \frac{1}{\lambda_{\text{base}}^\circ}.$$

Thus the same engagement factor appears reciprocally across the dual structural faces:

$$\lambda_x^\circ \propto \kappa_x^{-1}, \quad M'_x{}^\circ \propto \kappa_x.$$

In particular, hydrogen and neutron satisfy

$$\lambda_H^\circ = \frac{\lambda_p^\circ}{\sqrt{\pi}}, \quad M'_H{}^\circ \propto \sqrt{\pi},$$

and

$$\lambda_n^\circ = \frac{\lambda_p^\circ}{\pi}, \quad M'_n{}^\circ \propto \pi.$$

This suggests a Reciprocal Engagement Principle: admissibility engagement factors migrate inversely between the λ - and M' -faces while preserving the underlying closure invariant.

Squared Migration Rule in the KG Regime

Let

$$K_x := (2\pi M'_x{}^\circ)^2$$

denote the Klein–Gordon-type closure insertion for a configuration x .

If

$$\lambda_x^\circ = \frac{\lambda_p^\circ}{\kappa_x},$$

then

$$M'_x{}^\circ = \frac{\kappa_x}{\lambda_p^\circ},$$

and therefore

$$K_x = (2\pi M'_x{}^\circ)^2 = \kappa_x^2 (2\pi M'_p{}^\circ)^2.$$

Thus the admissibility engagement factor squares in the self-paired regime:

$$K_x = \kappa_x^2 K_p.$$

For hydrogen, with $\kappa_H = \sqrt{\pi}$,

$$K_H = \pi K_p.$$

For neutron, with $\kappa_n = \pi$,

$$K_n = \pi^2 K_p.$$

This indicates that self-pairing squares not only the closure insertion but also the engagement weighting.

HF Counting Rule for the Appearance of M'

Let

$$M' = \frac{1}{\lambda}$$

denote inverse closure length.

Within the LMR structural grammar, the appearance of M' depends on how admitted closure is counted.

1. **Single presentation.** A closure inserted once into a local admissibility condition gives

$$\mathcal{C}_1 \sim 2\pi M'.$$

2. **Self-paired presentation.** A closure evaluated against itself gives

$$\mathcal{C}_2 \sim (2\pi M')^2.$$

3. **Distributed presentation.** A closure presented across an embedding measure gives

$$\mathcal{C}_3 \sim \frac{M'}{V}.$$

Thus the exponent and placement of M' track admissibility context rather than introducing distinct mass primitives.

Structural Unification Reading (Fenced)

Once the HF counting rule is established, the reduced quantum and gravitational insertions may be read as distinct admissibility presentations of the same primitive

$$M' = \frac{1}{\lambda}.$$

On this reading:

$$2\pi M' \leftrightarrow \textit{singlelocalclosurepresentation},$$

$$(2\pi M')^2 \leftrightarrow \textit{self-pairedlocalclosurepresentation},$$

$$\frac{M'}{V} \leftrightarrow \textit{distributedclosurepresentation}.$$

Thus the reduced quantum and gravitational frameworks need not be interpreted as dimensionally incompatible descriptions. They may instead be read as distinct structural presentations of a shared closure primitive.

No claim of full mathematical unification is made here.

Three-Regime Classification of Reduced Mass Insertions

The HF counting rule identifies three admissibility contexts for the primitive

$$M' = \frac{1}{\lambda}.$$

These contexts correspond to distinct structural presentations of closure and are directly reflected in standard reduced equations.

Regime	Form	Structural Role	Representative Equation
Single closure	$2\pi M'$	Single local insertion	Dirac equation
Self-paired closure	$(2\pi M')^2$	Closure against itself	Klein–Gordon equation
Distributed closure	$M'V$	Closure per support measure	Dust stress-energy

In the Dirac equation,

$$(i\gamma^\mu \partial_\mu - 2\pi M')\psi = 0,$$

the mass term appears linearly, corresponding to a single closure insertion.

In the Klein–Gordon equation,

$$(+4\pi^2 M'^2)\phi = 0,$$

the mass term appears quadratically as $(2\pi M')^2$, corresponding to self-paired closure.

In contrast, the dust stress-energy tensor reduces as

$$T'_{\mu\nu} = \frac{M'}{V} u_\mu u_\nu,$$

in which closure appears linearly but is distributed across an embedding measure.

These three forms are not independent constructions. They arise from the same primitive M' under different admissibility contexts:

$$\begin{aligned} 2\pi M' &\leftrightarrow \textit{singlelocalpresentation}, \\ (2\pi M')^2 &\leftrightarrow \textit{self-pairedpresentation}, \\ \frac{M'}{V} &\leftrightarrow \textit{distributedpresentation}. \end{aligned}$$

Thus the distinction between linear and quadratic mass dependence, as well as the appearance of density-like terms, is traced to how closure is counted rather than to different underlying primitives.

No dynamical interpretation is implied; this is a structural classification within the reduced correspondence.

Structural Reading of the Three Regimes (Fenced)

The classification above suggests that the different appearances of M' may be read as distinct admissibility presentations of the same closure primitive.

This observation may be summarized as follows:

$$\begin{aligned} 2\pi M' &\leftrightarrow \textit{singlelocalclosurepresentation}, \\ (2\pi M')^2 &\leftrightarrow \textit{self-pairedclosurepresentation}, \\ \frac{M'}{V} &\leftrightarrow \textit{distributedclosurepresentation}. \end{aligned}$$

On this reading, the exponent and placement of M' do not introduce distinct mass primitives, but instead reflect how closure is counted within a given admissibility context.

In particular:

- Linear appearance corresponds to a single admitted closure inserted once into a local condition.
- Quadratic appearance corresponds to closure evaluated against itself.
- Density-like appearance corresponds to closure distributed across an embedding or support measure.

This interpretation is consistent with the reduced forms of the Dirac equation, Klein–Gordon equation, and simple stress-energy models.

However, no claim is made here that these structural regimes fully derive the corresponding physical theories. The present observation is classificatory and suggestive, not a completed derivation.

First-Order and Second-Order HF Admissibility

Let

$$M' = \frac{1}{\lambda}$$

denote inverse closure length.

A first-order admissibility condition tests the closure primitive once and therefore admits a linear insertion:

$$\mathcal{A}_1 \sim M'.$$

When closure is represented in cycle-dressed form, this becomes

$$\mathcal{A}_1 \sim 2\pi M'.$$

A second-order admissibility condition tests closure against itself and therefore admits a quadratic insertion:

$$\mathcal{A}_2 \sim (M')^2.$$

In cycle-dressed form, this becomes

$$\mathcal{A}_2 \sim (2\pi M')^2 = 4\pi^2 M'^2.$$

On this reading, the reduced Dirac and Klein–Gordon mass insertions may be interpreted as first-order and second-order admissibility presentations of the same closure primitive.

No claim is made here that the factor 2π has been derived structurally; it may reflect standard cycle/Fourier normalization in the observer-side formulation.

On the Status of 2π and 4π

Within the LMR grammar, 4π is already fixed as the minimal volumetric closure reference on the M' -face, while 2π is the natural circulatory closure factor associated with a loop-photon (LP) type closure.

In standard observer-side equations, however, factors of 2π also arise from cyclic/Fourier normalization. Accordingly, the appearance of 2π in reduced quantum equations may reflect either:

- a genuine closure residue of LP-type structure, or
- realization-layer packaging associated with cyclic representation.

This ambiguity does not apply equally to 4π , whose structural role in the LMR grammar is more sharply fixed.

Thus 2π should presently be treated as a candidate realization-layer closure marker, not yet as a uniquely structural invariant.

Two Roles of 2π

The appearance of 2π in standard equations must be separated into two distinct classes.

First, 2π arises generically from cyclic/Fourier representation, for example in

$$k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi f.$$

This is observer-side packaging and is not by itself structurally diagnostic.

Second, 2π survives ℓ_m -reduction in local mass insertions:

$$2\pi M', \quad (2\pi M')^2.$$

These factors remain after the bridge cancellation and therefore form a distinct reduced-mass residue.

The simplest distributed dust source,

$$\frac{M'}{V},$$

does not carry the same explicit 2π dressing.

This suggests that 2π -family residues track local closure presentation, whereas pure occupancy-density terms track distributed closure without the same cyclic dressing.

No claim is made here that all such residues are uniquely structural; some may still reflect observer-side cyclic packaging.

Provisional Closure Dressing Rule

In ℓ_m -reduced form, factors of the 2π -family appear selectively in expressions corresponding to local closure presentation, including:

- first-order insertion ($2\pi M'$),
- second-order self-pairing ($(2\pi M')^2$),
- localized field self-terms.

In contrast, distributed occupancy expressions such as M'/V do not carry explicit 2π -family factors.

This suggests that 2π -family residues track local closure presentation rather than distributed embedding density.

No claim is made that these factors are purely structural; they may partially reflect realization-layer cyclic packaging.

Closure Dressing Hypothesis

The factor 2π in reduced quantum equations may be interpreted as a realization-layer dressing associated with the presentation of a single closed circulation (LP-type closure).

Under this interpretation:

- $2\pi M'$ corresponds to a single closure insertion,
- $(2\pi M')^2$ corresponds to closure self-consistency (paired admissibility),
- absence of 2π corresponds to distributed occupancy without localized closure presentation.

Hydrogen Test of the 2π Closure Dressing

For hydrogen,

$$\lambda_H^\circ = \frac{\lambda_p^\circ}{\sqrt{\pi}} = \frac{4\pi}{\sqrt{\pi}\sqrt{c^4}}, \quad M_H'^\circ = \frac{1}{\lambda_H^\circ} = \frac{\sqrt{\pi}\sqrt{c^4}}{4\pi}.$$

Define the dual Dirac-style pair

$$D_H^{(M')} := 2\pi M_H'^\circ, \quad D_H^{(\lambda)} := 2\pi\lambda_H^\circ.$$

Then

$$D_H^{(M')} = 2\pi \cdot \frac{\sqrt{\pi} \sqrt{c^4}}{4\pi} = \frac{\sqrt{\pi} \sqrt{c^4}}{2},$$

and

$$D_H^{(\lambda)} = 2\pi \cdot \frac{4\pi}{\sqrt{\pi} \sqrt{c^4}} = \frac{8\pi^2}{\sqrt{\pi} \sqrt{c^4}}.$$

Thus the engagement factor $\sqrt{\pi}$ redistributes reciprocally across the two faces, while the invariant product remains

$$D_H^{(M')} D_H^{(\lambda)} = 4\pi^2.$$

This indicates that the factor 2π is not uniquely diagnostic of free LP closure. Rather, it behaves as a local closure dressing that survives across primitive and seated configurations, while admissibility engagement factors migrate reciprocally between the λ - and M' -faces.

Neutron Test of the 2π Closure Dressing

For neutron,

$$\lambda_n^\circ = \frac{\lambda_p^\circ}{\pi} = \frac{4}{\sqrt{c^4}}, \quad M_n'^\circ = \frac{1}{\lambda_n^\circ} = \frac{\sqrt{c^4}}{4}.$$

Define the dual Dirac-style pair

$$D_n^{(M')} := 2\pi M_n'^\circ, \quad D_n^{(\lambda)} := 2\pi \lambda_n^\circ.$$

Then

$$D_n^{(M')} = 2\pi \cdot \frac{\sqrt{c^4}}{4} = \frac{\pi \sqrt{c^4}}{2},$$

and

$$D_n^{(\lambda)} = 2\pi \cdot \frac{4}{\sqrt{c^4}} = \frac{8\pi}{\sqrt{c^4}}.$$

Their product remains invariant:

$$D_n^{(M')} D_n^{(\lambda)} = 4\pi^2.$$

Thus the 2π closure dressing survives even under torsion-locked pseudo-closure. This indicates that the factor 2π is not uniquely diagnostic of free LP structure, but behaves more generally as a realization-layer local closure dressing that persists across primitive, seated, and torsion-locked configurations.

The corresponding KG-style square is

$$K_n^{(M')} = \left(D_n^{(M')} \right)^2 = \frac{\pi^2 c^4}{4},$$

which confirms that self-pairing squares the engagement weighting in the same way it squares the closure insertion.

Face-Selection Rule for 2π and 4π

Within the LMR grammar, the factors 2π and 4π appear to belong to different closure scopes.

The factor

$$4\pi$$

is the natural marker of full volumetric closure and therefore serves as the closure reference on the λ -/ M' -face scaffold.

By contrast, the factor

$$2\pi$$

appears in reduced local insertions such as

$$2\pi M', \quad (2\pi M')^2,$$

and is therefore read most naturally as a local closure presentation or realization-layer dressing.

On this reading,

$$4\pi \leftrightarrow \text{closure reference}, \quad 2\pi \leftrightarrow \text{closure presentation}.$$

This distinction is preserved across primitive, seated, and torsion-locked configurations, while admissibility engagement factors redistribute reciprocally between the λ - and M' -faces.

No claim is made here that all appearances of 2π in standard formalisms are uniquely structural; some may still reflect observer-side cyclic packaging.

Complementary-HF Reading of the 2π Dressing

A stronger structural reading of the factor 2π is obtained by interpreting the coefficient 2 as the count of complementary half-folds required for a minimal closed circulation.

On this reading,

$$2\pi$$

does not merely mark generic cyclic packaging, but a local realized closure formed by two complementary half-folds.

This gives the reduced Dirac insertion

$$2\pi M'$$

the interpretation of a single complementary-HF closure presentation of the primitive $M' = 1/\lambda$.

The corresponding Klein–Gordon insertion

$$(2\pi M')^2$$

then represents self-pairing of that same complementary-HF closure presentation.

By contrast, distributed occupancy terms such as

$$\frac{M'}{V}$$

carry no explicit 2π dressing and therefore do not present closure in the same local complementary-HF form.

Within this hierarchy,

$$4\pi$$

remains the full closure reference, while

$$2\pi$$

marks local complementary-HF closure presentation.

Speculative Link to Measurement

The factor 2π appears selectively in reduced expressions corresponding to local closure presentation, while being absent from distributed occupancy terms.

This suggests a possible structural interpretation in which measurement corresponds to the enforcement of complementary-HF closure, transitioning a distributed configuration into a locally admissible closure state.

Under this interpretation, the 2π factor would serve as a marker of local closure realization rather than purely cyclic representation.

This connection remains speculative and is not asserted as a dynamical mechanism.

Closure Interpretation of Particle-in-a-Box Quantization

For a one-dimensional domain of length L , the standard condition

$$k_n = \frac{n\pi}{L}$$

combined with $k = 2\pi/\lambda$ yields

$$\lambda_n = \frac{2L}{n}.$$

This condition may be read structurally as requiring that the domain admit an integer number of complementary half-fold segments:

$$L = \frac{n}{2}\lambda_n.$$

The explicit 2π factor does not appear in the final expression because it is absorbed into the global boundary constraint.

This contrasts with local insertion expressions such as $2\pi M'$, where the same factor appears explicitly as a marker of local closure presentation.

Accordingly, global boundary quantization enforces complementary-HF closure at the domain level, while local insertion expressions enforce closure at a point or localized configuration.

Harmonic Oscillator as Laddered Half-Fold Admissibility

The quantum harmonic oscillator spectrum

$$E_n = \hbar\omega \left(n + \frac{1}{2} \right)$$

reduces under ℓ_m -substitution to

$$E'_n = \frac{E_n}{\ell_m} = \frac{c\omega}{2\pi} \left(n + \frac{1}{2} \right).$$

Unlike the Dirac and Klein–Gordon mass insertions, the oscillator does not primarily display an explicit local 2π closure insertion. Instead, its discrete structure is carried by the factor

$$n + \frac{1}{2}.$$

This suggests a distinct admissibility regime in which quantization arises from a ladder of allowed states built upon an irreducible half-step.

Within the LMR grammar, this half-step is naturally suggestive of a persistent half-fold baseline rather than full complementary closure.

Accordingly, the harmonic oscillator may be read not as a boundary-fit closure problem, but as a laddered admissibility problem with an irreducible half-fold remainder.

Table 1: Occurrence of 2π -family factors across reduced forms

Form	Local vs Distributed	2π-family present?	Candidate Interpretation
$2\pi M'$	Local insertion	Yes	Single closure dressing (first-order)
$4\pi^2 M'^2 = (2\pi M')^2$	Local self-pairing	Yes	Paired closure dressing (second-order)
$2\pi^2 M'^2 \phi^2$	Local field self-term	Yes	Localized field self-coupling
$M'V$	Distributed density	No (explicit)	Occupancy / embedding density (non-local)